In the data fusion approach, independent vertical profiles are retrieved from the observations of two or more instruments that observe the same portion of the atmosphere in different spectral region or with different geometries and subsequently combined through a specific algorithm into a single estimate.

We present here the new data fusion method used in the project to combine the information associated to the operational products of the LEO instruments, as well as to the ones on the GEO mission.

We suppose to have N independent simultaneous measurements of the vertical profile of an atmospheric species referred to the same geolocation.

- Performing the retrieval of the N measurements we obtain N vectors \( \hat{x}_i \) (i=1, 2, ..., N) that provide independent estimates of the profiles on a common vertical grid. These vectors are characterized by the covariance matrices (CM) \( S_i \) and the averaging kernels (AKM) \( A_i \).

- Exploiting the definition of AKM, we can expand at the first order the relationship that exists between retrieved profile and true profile:

\[
\hat{x}_i = x_{\text{true}} + A_i (x_{\text{true}} - x_{\text{pre}}) + \sigma_i
\]

where \( x_{\text{true}} \) is a priori profile used in the i-th retrieval, \( x_{\text{true}} \) is true profile, \( \sigma_i \) is errors on \( \hat{x}_i \).

Rearranging we obtain:

\[
\alpha_i = \hat{x}_i - (I - A_i) x_{\text{true}} = A_i x_{\text{true}} + \sigma_i
\]

Since the vectors \( \alpha_i \) are measurements of \( x_{\text{true}} \), we can perform a simultaneous fit of these measurements minimizing the following cost function:

\[
c(x) = \sum_{i=1}^{N} (\alpha_i - A_i x_{\text{true}})^{T} S_i^{-1} (\alpha_i - A_i x_{\text{true}}) + (x_{\text{true}} - x_{\text{pre}})^{T} S^{-1} (x_{\text{true}} - x_{\text{pre}}).
\]

\( x_{\text{true}} \) and \( S_i \) are the a priori profile and its CM that we want to use as a constraint of the fit.

The minimum of \( c(x) \) is obtained for:

\[
x_{\text{true}} = \left[ \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i + S^{-1} \right]^{-1} \sum_{i=1}^{N} A_i^{T} S_i^{-1} \alpha_i + S^{-1} x_{\text{pre}}.
\]

This relationship provides a new estimate of the profile determined with the data fusion of \( N \) different profiles.

This fused profile has a CM, obtained propagating the errors of \( \alpha_i \) into \( x_{\text{true}} \), equal to:

\[
S = \left[ \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i + S^{-1} \right]^{-1} \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i + S^{-1} \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i + S^{-1}.
\]

and an AKM, obtained performing the derivative of \( x_{\text{true}} \) with respect to the true profile, equal to:

\[
A = \left[ \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i + S^{-1} \right]^{-1} \sum_{i=1}^{N} A_i^{T} S_i^{-1} A_i.
\]